

Meson Exchange Effect on Color Superconductivity

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Abstract

We investigate the effects of pion and gluon exchanges on the formation of two-flavor color superconductivity at moderate density, $\mu < 1\text{GeV}$. The chiral quark model proposed by Manohar and Georgi containing pions as well as gluons is employed to show that the pion exchange reduces substantially the value of the superconducting gap gotten with the gluon exchange only. It turns out that the pion exchanges produce a repulsion between quark-quark pair in a spin and isospin singlet state. We suggest that the phase consisting of pions, gluons and quarks is one of the candidates of in-medium QCD phase at moderate density.

1 Introduction

Due to the attractive interactions between two quarks from one-gluon exchange or 't Hooft interaction induced by instanton, there is a tendency toward spontaneous breaking of (color) gauge symmetry forming color superconductivity and especially color-flavor-locking(CFL) [1][2]. The magnitude of the superconducting gap Δ is estimated by using models whose parameters are chosen to reproduce reasonable zero density physics [1][2] or from the perturbative one-gluon exchange calculations which are valid at asymptotically high density[3][4]. Recently, it is shown that at intermediate densities $\mu \sim 500 MeV$, large enough for the system to be in the quark phase but small enough to support nonperturbative interactions, Cooper pairs with nonzero total momentum are favored leading to gaps which vary in space[5]. For recent reviews on the color superconductivity in quark matter, see Ref. [6].

On the other hand, the dense nuclear matter has been successfully described in terms of the hadrons, which will be called *hadronic* phase. It was already conceived that the boson condensation, particularly kaon condensation, can be produced at the relatively low density, about 3-4 times of the normal nuclear matter density. Indeed, the hadrons as the quasi-particles are subdued to the pattern of BR (Brown-Rho) scaling[7], whose relation to the results based on the conventional hadronic many-body calculation[8] is investigated in Ref.[9].

Between the hadronic and quark phases, we could expect a phase whose relevant degrees of freedom are mesons as Goldstone bosons of spontaneous breaking of chiral symmetry by non-zero value of chiral order parameter $\langle\bar{\psi}\psi\rangle$ and gluons due to lack of confinement as well as constituent (quasi) quarks[10], and we will call this phase as the CQ(constituent or chiral quark) phase. Phase diagram for QCD involving hadronic(nuclear) matter and quark matter(2SC, CFL) is conjectured and depicted in [11]. We propose that the CQ phase is one of the candidates of in-medium QCD phase at moderate density. A recent study on baryons in matter-free space reveals that the ground states and excitation spectra of light and strange baryons is governed by the Goldstone boson exchange(GBE) as well as the harmonic confinement potential between constituent quarks (quasi-quarks) [12]. Based on the decoupling of vector interactions at high density, it is argued in Ref. [13] that quasi-particle picture involving (quasi) quarks becomes more appropriate at higher densities as chiral restoration is approached.

The goal of this work is to study two-flavor color superconductivity (2SC) in the CQ phase and therefore to investigate a phase boundary of hadronic matter and quark matter (especially 2SC). We calculate the gap of two-flavor color superconductivity in the CQ phase using the chiral quark model proposed by Manohar and Georgi[10]. Note that the model [10] is valid around the energy scale E_{CQ} given in matter-free space $\Lambda_{QCD} < E_{CQ} < \Lambda_{\chi SB} \sim 1 GeV$. We expect, however, that the model is relevant to describe QCD at moderate density since we can argue that $\mu \sim E_{CQ}$ based on the renormalization group analysis at finite density[14].

In section 2, we introduce the chiral quark model and derive a gap equation for the Cooper pair from one gluon and one pion exchange. To see the physics concealed in the gap equation economically, we adopt contact four-Fermi interactions, which are assumed to reproduce the physics of one gluon and one pion exchange, and solve the gap equation in section 3. Discussions are presented in section 4.

2 Schwinger-Dyson equation in the Chiral Quark Model

The chiral quark model is defined between $\Lambda_{QCD} \sim 200 MeV$ and $\Lambda_{\chi SB} \sim 1 GeV$ and therefore contains mesons as Goldstone bosons of $SU(3)_L \times SU(3)_R$ spontaneous chiral symmetry breaking. And gluons as gauge bosons of $SU(3)_{color}$ should be also present in this phase of deconfinement. The chiral quark lagrangian is given by

$$\begin{aligned} \mathcal{L} = & \bar{\psi}(i\mathcal{D} + \mathcal{V})\psi + g_A \bar{\psi} \mathcal{A} \gamma_5 \psi - m \bar{\psi} \psi + \mu \bar{\psi} \gamma^0 \psi \\ & + \frac{1}{4} f_\pi^2 tr(\partial^\mu \Sigma^\dagger \partial_\mu \Sigma) - \frac{1}{2} tr(F^{\mu\nu} F_{\mu\nu}) + \dots \end{aligned} \quad (1)$$

where

$$\begin{aligned} D_\mu &= \partial_\mu + ig G_\mu, \quad G_\mu = G_\mu^a T^a, \\ V_\mu &= \frac{i}{2} (\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger), \\ A_\mu &= \frac{i}{2} (\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger), \\ \xi &= e^{(i\Pi/f_\pi)}, \quad f_\pi \simeq 93 \text{ MeV}, \quad \Sigma = \xi \xi \end{aligned}$$

and

$$\Pi = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}. \quad (2)$$

Note that in the chiral quark model, *we can establish a power counting argument to deal with non-renormalizable terms and to neglect internal gluon lines* [10].

Since the chiral quark model is valid below the chiral symmetry breaking scale, the value of the chiral order parameter $\langle \bar{\psi} \psi \rangle$ is not zero. We expect, however, the value of $\langle \bar{\psi} \psi \rangle$ in CQ phase to be quite small compared to that in the matter-free space. To see the effects of GBE on color superconductivity, we consider one pion exchange as well as one gluon exchange with $N_f = 2$. As it is described in the conjectured phase diagram for QCD at zero temperature [17], when chemical potential exceeds the constituent strange quark mass the more relevant phase will be $2SC + s$ in which we have ss condensates as well as Cooper pairs composed of u and d quarks. We can, however, neglect the difference between the $2SC$ and $2SC + s$ since the ss condensate is expected to be small [17]. In this work, we ignore ss condensate and concentrate on $2SC$. Since the value of chiral order parameter is assumed to be small, we may take the light chiral quark masses to be negligible compared to chemical potential. There are several works on the possibility of mixed phase of both chiral condensate and color superconductivity[18].

In this work, we use the Nambu-Gorkov formalism with

$$\Psi \equiv \begin{pmatrix} \psi \\ \bar{\psi}^T \end{pmatrix} \equiv \begin{pmatrix} \psi \\ \psi_c \end{pmatrix}.$$

The inverse quark propagator * is

$$S^{-1}(q) = \begin{pmatrix} \not{q} + \mu\gamma_0 - m & \bar{\Delta} \\ \Delta & (\not{q} - \mu\gamma_0 + m)^T \end{pmatrix} \quad (3)$$

where $\bar{\Delta} = \gamma_0 \Delta^\dagger \gamma_0$. To leading order in the perturbative expansion, the quark-gluon vertex matrix is $-ig\Gamma_\mu^a$ and the quark-pion vertex matrix is given by $-(g_A/f_\pi)\Gamma^i(q)$.

$$\begin{aligned} \Gamma_\mu^a &= \begin{pmatrix} T^a \gamma^\mu & 0 \\ 0 & -(T^a \gamma^\mu)^T \end{pmatrix} \\ \Gamma^i(q) &= \begin{pmatrix} \frac{\tau^i}{2} \not{q} \gamma_5 & 0 \\ 0 & -(\frac{\tau^i}{2} \not{q} \gamma_5)^T \end{pmatrix} \end{aligned} \quad (4)$$

The gluon propagator in the hard dense loops (HDL) approximation is given by

$$D_{\mu\nu}(q) = \frac{P_{\mu\nu}^T}{q^2 - G} + \frac{P_{\mu\nu}^L}{q^2 - F} - \xi \frac{q_\mu q_\nu}{q^4} \quad (5)$$

where

$$\begin{aligned} P_{ij}^T &= \delta_{ij} - \hat{q}_i \hat{q}_j, \quad P_{00}^T = P_{0i}^T = 0 \\ P_{\mu\nu}^L &= -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} - P_{\mu\nu}^T \end{aligned}$$

For $q_0 \ll \vec{q} \rightarrow 0$ and to leading order in perturbation theory we have,

$$F = 2m^2, \quad G = \frac{\pi}{2} m^2 \frac{q_0}{|\vec{q}|},$$

with $m^2 = N_f g^2 \mu^2 / (4\pi^2)$. The pion propagator is $D(k) = 1/k^2$.

The Schwinger-Dyson equation for gap matrix Δ becomes,

$$\begin{aligned} \Delta(k) &= ig^2 \int \frac{d^4 q}{(2\pi)^4} (-T^a \gamma^\mu)^T S_{21}(q) (T^b \gamma^\nu) D_{\mu\nu}(q - k) \\ &\quad + i(\frac{g_A}{f_\pi})^2 \int \frac{d^4 q}{(2\pi)^4} (-\frac{\tau^i}{2} (\not{q} - \not{k}) \gamma_5)^T S_{21}(q) (\frac{\tau^i}{2} (\not{k} - \not{q}) \gamma_5) \delta^{ij} D(q - k) \end{aligned}$$

Let us take the form of the gap matrix as [3]

$$\Delta_{ij}^{ab}(q) = (\lambda_2)^{ab} (\tau_2)_{ij} C \gamma_5 [\Delta_1(q_0) \frac{1 + \vec{\alpha} \cdot \hat{q}}{2} + \Delta_2(q_0) \frac{1 - \vec{\alpha} \cdot \hat{q}}{2}] \quad (6)$$

where $\vec{\alpha} = \gamma_0 \vec{\gamma}$. By inverting the inverse quark propagator matrix $S^{-1}(q)$, we obtain the 21-component of $S(q)$

$$S_{21}(q) = -(\lambda_2 \tau_2 C \gamma_5) \left(\frac{\Delta_1}{q_0^2 - (|\vec{q}| - \mu)^2 - \Delta_1^2} \Lambda_- + \frac{\Delta_2}{q_0^2 - (|\vec{q}| + \mu)^2 - \Delta_2^2} \Lambda_+ \right)$$

*Note that in the 2SC phase Cooper pairs of quarks cannot be color singlets and pick a color direction, the 3 direction for example. Therefore the color 3 quarks remain ungapped.

where $\Lambda_+ = (1 + \vec{\alpha} \cdot \hat{q})/2$ and $\Lambda_- = (1 - \vec{\alpha} \cdot \hat{q})/2$. For color and isospin, we can use the following relation,

$$\frac{1}{4}(\lambda_a)^T \lambda_2 \lambda_a = -\frac{N+1}{2N} \lambda_2 = -\frac{2}{3} \lambda_2, \quad (N_c = 3) \quad (7)$$

where we have used $(\lambda^a)_{ij}(\lambda^a)_{kl} = -2/N \delta_{ij} \delta_{kl} + 2 \delta_{il} \delta_{jk}$. For isospin $N_f = 2$, we use $\frac{1}{4}(\tau_i)^T \tau_2 \tau_i = -\frac{3}{4} \tau_2$. Then the gap equation becomes

$$\begin{aligned} \Delta(k) &= \lambda_2 \tau_2 C \gamma_5 (\Delta_1(k_0) \Lambda_+ + \Delta_2(k_0) \Lambda_-) \\ &= i g^2 \left(+\frac{2}{3} \lambda_2 \tau_2 C \gamma_5 \right) \int \frac{d^4 q}{(2\pi)^4} \gamma_\mu \left(\frac{\Delta_1}{q_0^2 - (|\vec{q}| - \mu)^2 - \Delta_1^2} \Lambda_- \right. \\ &\quad \left. + \frac{\Delta_2}{q_0^2 - (|\vec{q}| + \mu)^2 - \Delta_2^2} \Lambda_+ \right) \gamma_\nu D^{\mu\nu}(q - k) \\ &\quad + i \left(\frac{g_A}{f_\pi} \right)^2 \left(+\frac{3}{4} \lambda_2 \tau_2 C \gamma_5 \right) \int \frac{d^4 q}{(2\pi)^4} (\not{q} - \not{k}) \gamma_5 \left(\frac{\Delta_1}{q_0^2 - (|\vec{q}| - \mu)^2 - \Delta_1^2} \Lambda_- \right. \\ &\quad \left. + \frac{\Delta_2}{q_0^2 - (|\vec{q}| + \mu)^2 - \Delta_2^2} \Lambda_+ \right) (\not{q} - \not{k}) \gamma_5 D(q - k) \end{aligned} \quad (8)$$

Following the same procedure shown in Ref. [3], we get the gap equation,

$$\begin{aligned} \Delta(k_0) &= \frac{g^2}{18\pi^2} \int_0^\infty dq_0 \frac{\Delta(q_0)}{\sqrt{q_0^2 + \Delta^2(q_0)}} \ln \left(\frac{b\mu}{|k_0 - q_0|} \right) \\ &\quad - \frac{3\mu^2}{16\pi^2} \frac{g_A^2}{f_\pi^2} \int_0^\infty dq_0 \frac{\Delta(q_0)}{\sqrt{q_0^2 + \Delta^2(q_0)}} \frac{(k_0 - q_0)^2}{2\mu^2} \ln \left(\frac{4\mu^2}{|k_0 - q_0|^2} \right). \end{aligned} \quad (9)$$

From (9) we can see that one pion exchange gives repulsive interaction in the quark-quark potential and therefore the magnitude of the gap will be reduced in the presence of the GBE. Note that the quark pairs are in isosinglets and spin singlet state. The reason that one pion exchange(OPE) gives repulsive potential in the quark-quark potential can be understood from the similarity with nucleon-nucleon potential from OPE in the same isosinglet and spin singlet channel, which is repulsive[16]

$$V_\pi(S = I = 0) = \frac{3f^2}{4\pi} \frac{e^{-m_\pi r}}{r}. \quad (10)$$

We note here that it is not simple to solve the gap equation (9) analytically even with the approximation given in [4].

3 A Toy Solution

The property of the gap equation (9) can be understood by a toy solution in which we set forth the following four Fermi interactions mimicking the quark-quark interaction in the chiral quark model. Note that with a NJL type interaction our gap will have different dependence on the coupling constant $\Delta \sim \exp(-1/g^2)$ from the one with one-gluon exchange $\Delta \sim \exp(-1/g)$ [4].

We can see, however, the effects of the GBE on the gap explicitly in a simple manner with the NJL type interaction.

$$\mathcal{L}_{int} = -G \int d^4x (\bar{\psi} \lambda^a \gamma^\mu \psi) (\bar{\psi} \lambda^a \gamma_\mu \psi) + G_\pi \int d^4x (\bar{\psi} \vec{\tau} \gamma_5 \psi) \cdot (\bar{\psi} \vec{\tau} \gamma_5 \psi) \quad (11)$$

where the first term represents gluon induced interaction, and the second one is pion induced interaction. Here λ^a is a generator of $SU(3)$ color group and $\vec{\tau}$ is for $SU(2)$ isospin group. Note that we choose positive sign in front of pion induced term considering repulsive OPE potential (9) and take G and G_π to be positive. Since pion is almost massless, it is not simple to put the one-pion exchange into the four-Fermi form, but here we expect that the four-Fermi interaction with G_π in (11) could at least mimic the *repulsive* nature of the one-pion exchange calculated in section 2.

Now it is straightforward to obtain a gap equation with the interacting Lagrangian (11) and we get

$$\Delta = \left(\frac{2G\mu^2}{3\pi^2} - \frac{3G_\pi\mu^2}{16\pi^2} \right) \int dq_0 \frac{\Delta}{\sqrt{q_0^2 + \Delta^2}} \quad (12)$$

Note that this gap equation can be traced from the equation (9) by removing propagator dependence, *i.e.* by neglecting logarithmic term in (9). For the sake of simplicity, we introduce cut-off Λ and take $\Lambda \sim \mu$. Since the gap Δ is constant with the interaction (11), we get a solution easily

$$\Delta = 2\mu \exp(-1/N) \quad (13)$$

with $N \equiv \frac{2G\mu^2}{3\pi^2} - \frac{3G_\pi\mu^2}{16\pi^2}$. Now we can see that due to the repulsive nature of pion exchange potential, the value of the gap will be reduced by a factor $\sim \exp[-1/(G_\pi\mu^2)]$.

To estimate the value of the gap, we take $G \approx 5.8 GeV^{-2}$ and

$$G_\pi = c \frac{g_A^2}{f_\pi^2} \quad (14)$$

where c is a constant. The value of c could be determined by fully integrating out pions to obtain effective action containing four-Fermi interactions, but we take it as a free parameter and we shall use $g_A = 0.75$ and $f_\pi = 93 MeV$ given in free space for numerical estimates.

Taking $G \approx 5.8 GeV^{-2}$ to reproduce the results from NJL model calculations with only single gluon exchange effects and choosing $c = 0.05 (G_\pi \approx 3.3 GeV^{-2})$, we get the results depicted in Fig. 1.

We note here that when the value of G_π is big enough to make N to be negative, we see from (12) that there is a possibility to have no color superconducting gap at all.

4 Discussion

In this work, we employ the chiral quark model to study the effect of one pion exchange on the two flavor color superconductivity. We found that OPE reduces the value of the gap Δ , though it is difficult to be quantitative as to how much the gap will be reduced since we have no definite information on the value of G_π . Our work will have several important consequences :

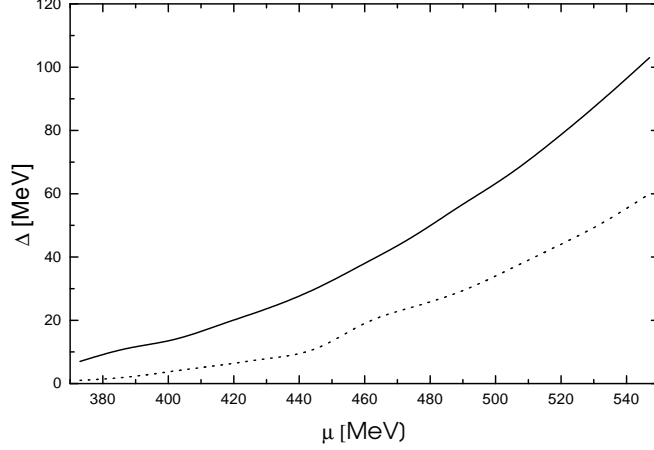


Figure 1: Δ as a function of chemical potential. Dotted line represent Δ with both one gluon and one pion exchange and solid line is the resulting gap from only one-gluon exchange contribution.

- At moderate density where CQ phase is plausible, pion as a Goldstone boson of spontaneous breaking of chiral symmetry by quark condensate $\langle\bar{\psi}\psi\rangle$ could do important role in the physics of color superconductivity, implying that at moderate density there could be some other interactions as well as one gluon exchange which have to be taken into account. Besides the GBE mode considered in this work, light vector mesons could come in as indicated by BR scaling [7] and the “vector manifestation” [19] in which the chiral symmetry is restored at the critical point by the massless degenerate pion and (longitudinal) ρ -meson as the chiral partner. In the case of two-nucleon states of total isospin 0 and spin 0, the ρ exchange potential is given by [16]

$$V_\rho(S = I = 0) = 3\left[-\frac{g_\rho^2}{4\pi} + 2\frac{f_\rho^2}{4\pi}\right]\frac{e^{-m_\rho r}}{r} \quad (15)$$

where $f_\rho = (g_\rho + g_t)(m_\rho/2m_N)$. When we take the value of the couplings determined in free space, we can easily see that the ρ exchange potential is repulsive. Therefore we expect *naively* that light vector mesons also produce repulsion between quark-quark pair we are considering. We note, however, that the potential becomes attractive when neglecting tensor coupling ($g_t = 0$) and that density dependence of g_ρ and g_t should be properly taken into account. In the vector manifestation, we should also keep it in mind that the transverse ρ is decoupled from the vector current.

- The effects of diquark condensates on the cooling of compact stars is investigated by several authors, see for example [20]. However it is expected that the quark matter core in neutron star is *inert* as far as cooling is concerned since the quasi-quarks contribution to the specific heat and neutrino emissivity is suppressed by a factor $\sim \exp(-\Delta/T)$ [21]. However, when GBE reduces the gap substantially, we expect that quark matter core could play important role in cooling of neutron star and the formation of neutron star [22].

- Extending our work to three-flavor QCD will be quite interesting. For example, flavor non-singlet nature of the GBE may induce mixing between LL and RR condensates which is not possible with only gluon exchange interactions and reduction in the value of the gap will

affect meson masses in CFL phase [23] and the physics of kaon condensation in quark matter [24], which will be reported elsewhere.

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